THE FAILURE LIMIT AND THE STRENGTH CONDITIONS OF A SOIL

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A form of strength condition is discussed for a material that hardens on deformation, the case being that of complex loading. Test results are discussed for sands and loams subject to triaxial compression. It is shown that the characteristics of the parts must appear in the strength condition for some complex loading paths. The shortening of the failure curve for these varieties of soil is determined by the length of the arc on the surface of the loading sphere.

Postulates of continuity, uniformity, and isotropic behavior are involved in the mechanics of deformable continua, and it is assumed that the strength is completely determined by the stresses arising at points in the medium. The transition to the yield state is examined in relationship to three parameters, as which one usually takes either the three principal stresses or the three invariants of the stress tensor [1, 2].

Some strength criteria relate to the case of simple or proportional loading; the difficulties arising in constructing the failure surface are associated with the effects of the loading history. The series used in soil mechanics as strength criteria do not take into account the behavior during deformation preceding failure. The details of plastic deformation in a soil vary with the loading path, and the effects on the current and final parameters are substantial, which itself is reflected in the magnitude of the failure stress. Test results with steel [3] show that the failure limit is substantially dependent on the path; the failure curve lies within the curve obtained with simple loading.

It is of interest to formulate the strength criterion for a work-hardening material with allowance for the details of the loading preceding the limiting state. One assumes that such a condition will be dependent on the invariant characteristics of the isothermal loading, i.e., the geometrical properties such as the arc length and modulus of the stress vector. Paths with kinks are of interest for soil loading, since these characterize combined stresses with unvarying stress intensity [4]. A form of this strength condition is described below.

We use a geometrical representation of loading [5]. We represent the stress tensor σ_{ij} in five-dimensional space by the vector S, while the loading is represented by the motion of the end of the vector along the loading path. An element of arc is defined by

$$dT = (dS_1^2 + dS_2^2 + \ldots + dS_5^2)^{1/2} \tag{1}$$

where $dS_1, dS_2, \ldots, dS_5$ are vector elements on the loading arc.

The expression for the modulus of S takes the form

$$|S| = P = \frac{1}{\sqrt{3}}[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2)]^{1/2} \tag{2}$$

where $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}$ are components of the stress tensor.

We use the following expression for the general case of loading with a kink in the path for a soil:

$$F(\sigma, \sigma_i, \mu) + \Psi(P, T) = 0 \tag{3}$$

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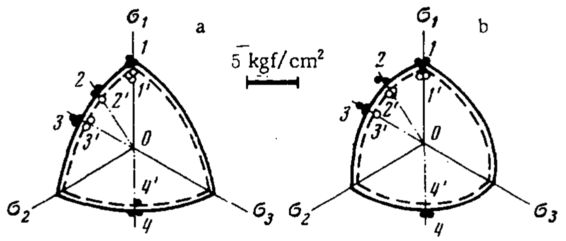

Fig. 1

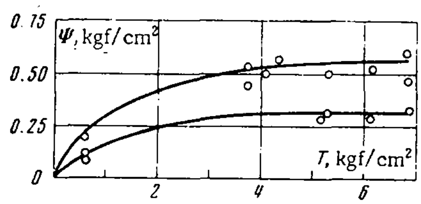

Fig. 2

The first term in (3) is a function of the three invariants σ_{ij}, which characterizes the limiting stress for the soil for simple or nearly simple loads [5]. The second term describes the strength change in relation to the modulus of the stress vector and the path length for combined loading. It is assumed that the mean pressure σ is given, while the shape of the curve on the surface of the loading sphere does not influence Ψ.

The boundary for failure stresses can be constructed by using tests on combined states of stress with detailed realizable parameters for the load; experiments on triaxial compression of sand and loam enable one to detail the form of the functions F and Ψ in (3).

The triaxial stress was produced by loading a hollow cylindrical specimen (height 80 mm, diameters 36 and 60 mm) by an axial force, a torque, and hydrostatic pressure. The apparatus enabled one to specify one tangential stress and three normal stresses, while having provision for measuring the corresponding components of the strain tensor. The apparatus and the characteristics of the soils have been described [4]. The tests were done with a fixed value for the mean pressure, which was set at the start by hydrostatic compression of the specimen (for sand, $\sigma = 3$ and 5 kgf/cm^2, while for loam σ was 5 kgf/cm^2). The load was increased with steps, followed by stabilization of the creep deformation. It was assumed that the limiting state had been attained if the shear deformation occurred at a constant or increasing rate for unchanged stress. The changes in the specimen parameters were taken into account in processing the data.

The series of experiments consisted of three groups. In the first group, the tests were done with a constant value for the parameter $\mu = (2\sigma_2 - \sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)$ of -1, 0, or $+1$. In the second group, the stress σ_i increased from 0 to the limiting value and the form of the stressed state altered or else the two principal axes of σ_{ij} were rotated through $\pi/4$.

The path in the third group contained a section of combined loading at a constant σ_i; at this stage, the applied loads were chosen in such a way as to provide a preset combination and sequence of values for μ between -1 and 0, with rotation of the axes for the principal stresses σ_1 and σ_3 through $\pi/4$. The process involved motion of the end of vector S over a sphere whose radius was 3, 3.96, or 4.87 kgf/cm^2 for sand, while for the loam it was 3.96 kgf/cm^2. Before and after the range of combined stress, the specimen was used with unvarying μ of -1 or 0 and fixed positions of the principal axes of the σ_{ij}.

We found that the mode of variation in the stress determined the features of the strengthening [4] and also the stress at which failure occurred; the greatest strength was obtained with simple loading for $\mu = -1$, while the least value was obtained for $\mu = +1$. An intermediate value for the limiting σ_i corresponded to $\mu = 0$.

Following [6], we represent the invariant function F in the form

$$F = (\sigma_i)^* + (K + L\sigma)(\mu - G)\mu \quad (4)$$

where $(\sigma_i)^*$ is the limiting stress in a pure shear combined with hydrostatic pressure σ, while K, L, and G are experimental parameters.

Figure 1 shows in the space of the principal stresses σ_1, σ_2, and σ_3, the sections of the failure surface by the plane $\sigma = 5$ kgf/cm^2 for loam (a) and sand (b), these surfaces being constructed via (4) (solid lines). A section of the failure surface is an equilateral curvilinear triangle formed by curves convex with respect to the center of gravity. The simple loading paths are denoted by 01, 02, 03, and 04. The tests in the second group are shown by filled circles. In these cases, the soils failed at the same σ_i as in simple loading. In the third series, the failure stresses were reduced at most by 5-7% for the loam and by 10-12% for sand relative to simple loading. Points 1', 2', 3', and 4' correspond to these tests in Fig. 1.

We found that the value of Ψ, which characterizes the change in the failure limit, was not dependent on the form of the curve on the surface of the loading sphere; Fig. 2 shows the observed dependence of Ψ on the arc length T, where the upper line corresponds to sand and the lower one to loam. There is a nonlinear relationship between Ψ and T in both cases. The rate of change of Ψ falls as the arc length increases.

The following schemes illustrate the position of the failure boundary on deformation in accordance with a given program with different values for the modulus of the stress vector:

P	3	3.96	3.96	3.96	4.87	4.87
Ψ	−0.5	−0.6	−0.5	−0.55	−0.6	−0.55

where P and Ψ are in kgf/cm^2.

Above a certain value of P which corresponds to the linear part of the relationship between stress and strain, Ψ is uniquely determined by the path length on the surface of the loading sphere; one can use a piecewise-linear function of the following form to approximate the experimental results:

$$\Psi = T\sigma / (N + RT) \tag{5}$$

where N and R are parameters.

The broken line in Fig. 1 gives a section of the failure surface in accordance with (3) using (4) and (5).

If the material is capable of withstanding a tensile stress, this strength condition can be interpreted in the space of the principal stresses σ_1, σ_2, and σ_3 in terms of a limiting surface whose vertex does not coincide with the origin. In such a case, Ψ for $\sigma = 0$ even in principle cannot become zero, and the expression for it must have a form different from (5).

The following conclusion may be drawn. The failure limit for material under simple or nearly simple loading is determined by the state of stress attained and is independent of the geometrical characteristics of the loading path. Loading with kinks reduces the dimensions of the limiting surface, and the strength condition must include elements of the path.

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